Practical Relative Degree in SMC systems: Frequency Domain Approach

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Outline

1. Introduction
   - Problem Statement
2. Practical Relative Degree
3. Performance Margins
   - Definition of Performance Margins
   - Performance margins for conventional SMC
   - Performance margins for 2-SM (Twisting Algorithm)
4. Practical Relative Degree with Performance Margins Requirement
5. Conclusions

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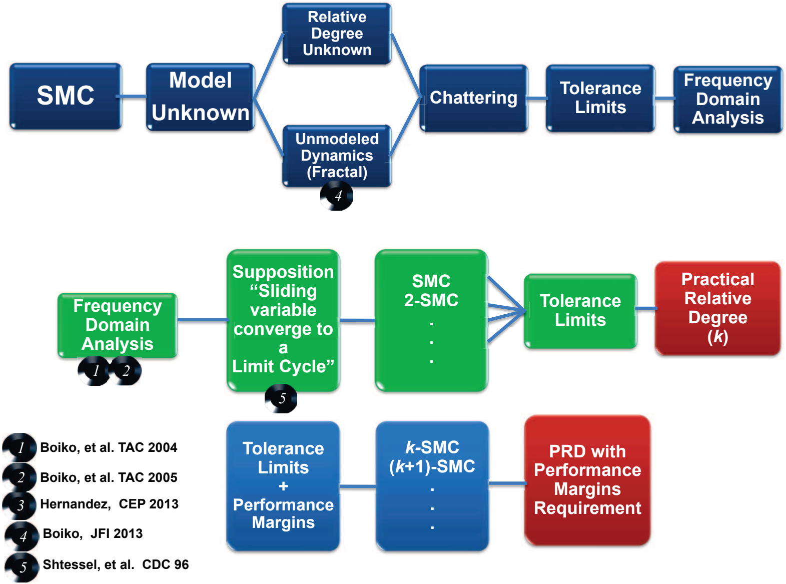
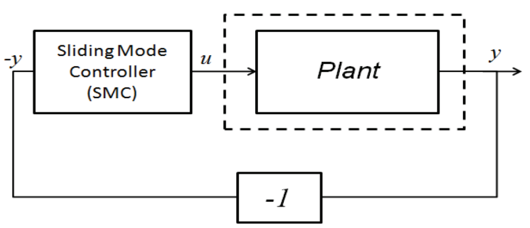
Introduction



Problem Statement

Control System



- A1. Plant has unknown relative degree r.
- A2. $W(j\omega)$ of the linear plant may be obtained.
- A3. Linear plant has low pass filter properties
- A4. Amplitude and phase frequency characteristics of $W(j\omega)$ are monotonously decreasing functions, i.e. $|W(j\omega_1)| > |W(j\omega_2)|$ and $\arg W(j\omega_1) > \arg W(j\omega_2)$ for $\omega_1 < \omega_2$
- A5. Describing Function of SMC may be obtained and it depends only on the amplitude A

Effects of Unmodeled Dynamics

Presence of unmodeled dynamics in the system increase the relative degree.

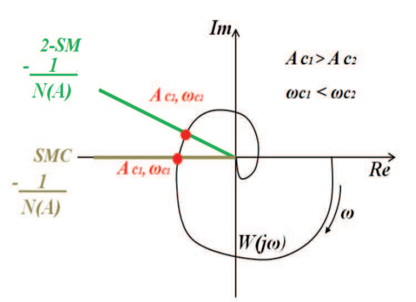
- The output y will not converge to zero but to a limit cycle [Shtessel, et al. 96],[Boiko, et al. 05]
- HB equation gives a solution and limit cycles in systems controlled by SMC may be predicted.

Tolerance Limits

The frequency ω_c and amplitude A_c are the tolerance limits of the acceptable limit cycle of the output y, so that self-sustained oscillations of the output y with the amplitudes $A \leq A_c$ and the frequencies $\omega \geq \omega_c$ yield the acceptable performance of the closed loop system in the real sliding mode [Utkin 09]

Motivation Example

Suppose system has $r > 2$ and it is controlled by SMC and 2-SMC



Case 3. Which controller is to be selected for the implementation?

Case\ Controller	SMC	2-SMC
Case 1	$A > A_c$ and/or $\omega < \omega_c$	$A > A_c$ and/or $\omega < \omega_c$
Case 2	$A > A_c$ and/or $\omega < \omega_c$	$A \leq A_c, \omega \geq \omega_c$
Case 3	$A \leq A_c, \omega \geq \omega_c$	$A \leq A_c, \omega \geq \omega_c$

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Practical Relative Degree

Practical Relative Degree (PRD) is understood as the smallest order r of SMC, that yields a predicted limit cycle in the closed-loop system with the amplitude $A \leq A_c$, $A_c > 0$ and the frequency $\omega \geq \omega_c$, $0 < \omega_c < \infty$.

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Performance Phase Margin

The Performance Phase Margin (PPM) in the system is the maximal additional phase shift in $W(j\omega)$ that the closed loop system can tolerate for its output y to exhibit the acceptable limit cycle with $A \leq A_c$, $A_c > 0$ and $\omega \geq \omega_c$, $0 < \omega_c < \infty$ in the real sliding mode.

Performance Gain Margin

The Performance Gain Margin (PGM) in the system is the maximum additional gain in $W(j\omega)$ that the closed loop system can tolerate for its output y to exhibit the acceptable limit cycle with $A \leq A_c$, $A_c > 0$ and/or $\omega \geq \omega_c$ $0 < \omega_c < \infty$ in the real sliding mode.

DF conventional SMC

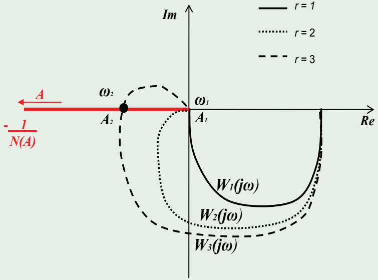
Let the controller be

$$u = -\alpha \mathrm{sign}(y) \tag{1}$$

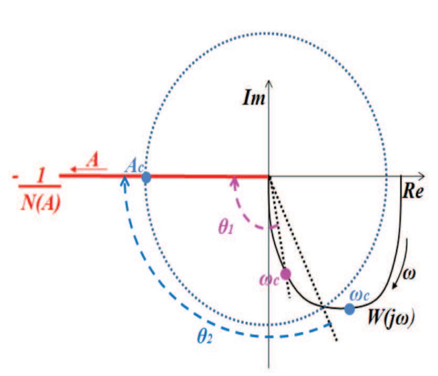
DF is $N(A) = 4\alpha/\pi A$, where A es the amplitude of y.
Harmonic Balance equation is

$$\mathrm{Re}\{W(j\omega)\} + j\,\mathrm{Im}\{W(j\omega)\} = -\frac{\pi A}{4\alpha} \tag{2}$$

where ω is the frequency of the output of system



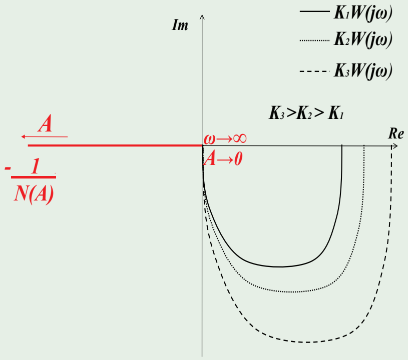
PPM SMC



Performance Phase Margin Method

- 1. Locate maximum amplitude A_c and the minimum frequency ω_c in $-1/N(A)$ and $W(j\omega)$, respectively.
- 3. Plot a circle with radio A_c.
  - 3(a) If the frequency ω_c is located outside of the circle, the PPM should be obtained as the angle formed between the intersection of the circle with $W(j\omega)$ and the negative real axis.
  - 3(b) If the frequency ω_c is located inside of the circle, the PPM should be obtained as the angle formed between the vector associated to ω_c and the negative real axis.

PGM SMC



Performance Gain Margin SMC

- Consider the gain $K > 1$ in the harmonic balance equation

$$K \cdot W(j\omega) = -\frac{1}{N(A)}$$

- Solve the harmonic balance equation for K with A_c and ω_c known,

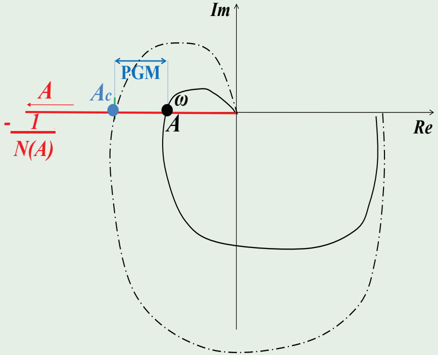
$$\begin{aligned} K\,\mathrm{Re}\,(W(j\omega)) &= -\frac{\pi A}{4\alpha} \\ K\,\mathrm{Im}\,(W(j\omega)) &= 0 \end{aligned}$$

- Value of $K \geq 1$ which satisfy HB equation system with $r \leq 2$ is $K \to \infty$

$$PGM \to \infty.$$

Performance Gain Margin Relay System

PGM Relay System



PGM Relay System

- 1. Consider the gain $K > 0$ in the harmonic balance equation

$$K \cdot W(j\omega) = -\frac{1}{N(A)}$$

- 2. Solve the harmonic balance equation for K with A_c and ω_c known,

$$K = -\frac{1}{N(A_c)}\frac{1}{W(j\omega_c)}$$

- 3. The value of K is the PGM of the system for the acceptable amplitude A_c.

Performance margins for 2-SM

DF for Twisting Algorithm

Let the controller be

$$u = -c_1 sign(y) - c_2 sign(\dot{y}) \tag{3}$$

where $c_1 > c_2 > 0$.
DF is

$$N(A) = (4/\pi A)(c_1 + jc_2),$$

where A is the amplitude of the output y
Harmonic Balance equation is

$$W(j\omega) = -\frac{A\pi}{4}\frac{c_1 - jc_2}{c_1^2 + c_2^2} \tag{4}$$

where ω is the frequency of the output y.

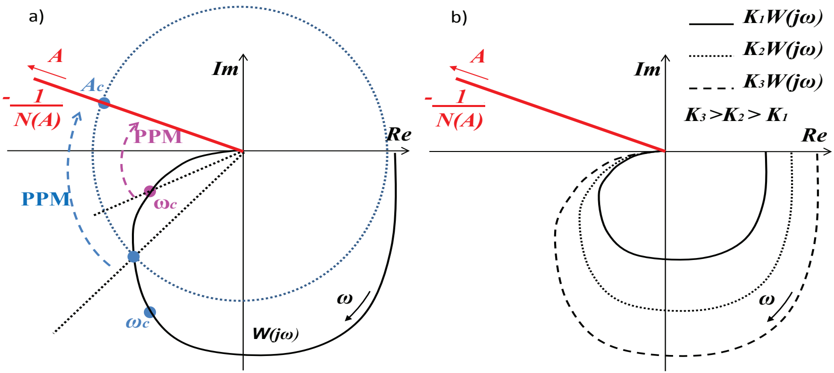


Figure: For Twisting Algorithm. a)PPM, b)PGM

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Suppose that PPM_c and PGM_c are the acceptable performance phase and gain margins.

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Therefore, the controller, which order corresponds to the calculated PRD, is to be implemented.

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- Practical relative degree in LTI SISO systems controlled by SMC is defined based on tolerance limits in terms of amplitude and frequency of a possible limit cycle on sliding variable.
- PRD is understood as the smallest order of SMC that yields acceptable performance.
- The proposed identification of practical relative degree can be used for the SMC design for systems treated as a black box.
- The notion is given in terms of performance margins which can be useful when the PRD could be affected by parameter changes or errors in the frequency response identification.

Conclusions