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Excerpt from the presentation

Fixed and Finite Time Stability in Sliding Mode Control

Arie Levant

School of Mathematical Sciences,
Tel-Aviv University, Israel
Homepage: http://www.tau.ac.il/~levant/

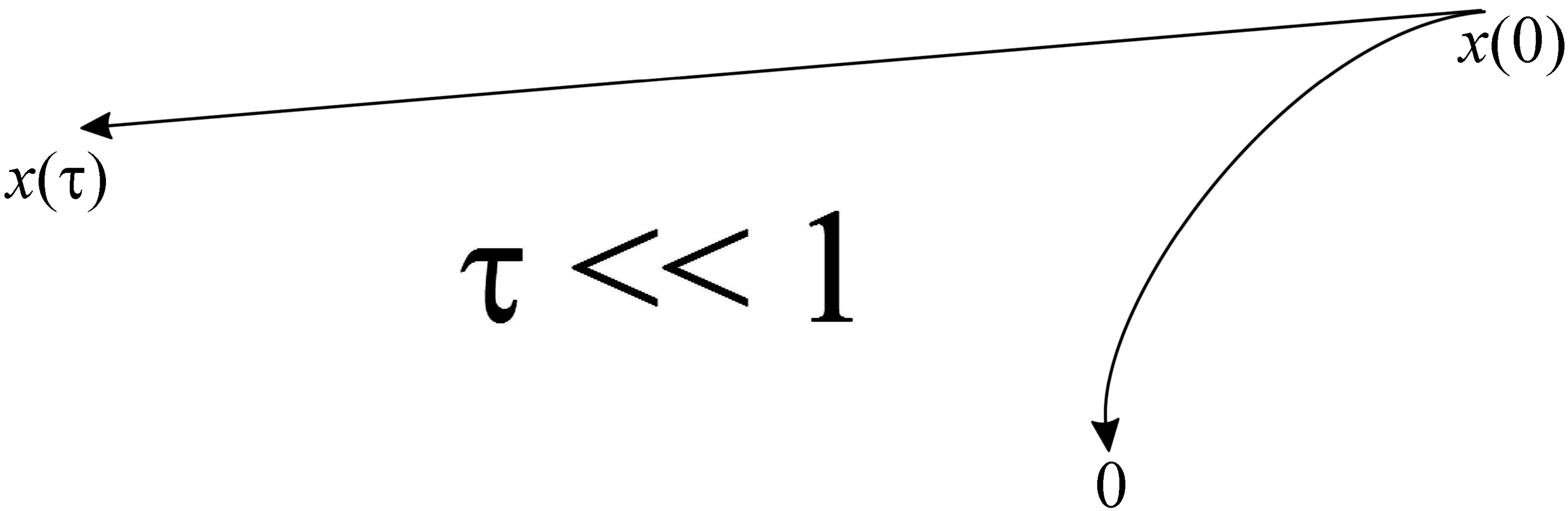
Feasibility of fixed-time stable control-1

$\dot{x} \in F(x)$ is fixed-time or practically fixed-time stable.

Proposition. $\forall \tau, \gamma > 0 \ \forall R > 0$
there are $x_0, \dot{x}_0, \ \ \| x_0 \| \geq R, \ \ \dot{x}_0 \in F(x_0),$ such that

$$\| \dot{x}_0 \| \tau \geq \gamma \| x_0 \|.$$

Take very small τ, and very large R and γ …



One Euler step can be incomparably larger than the current (arbitrarily large!) distance from the origin.

Any small delay (sampling step) is also not allowed.

τ-solutions (Euler approximations)

$\dot{x} \in F(x),\ \ x \in \mathbf{R}^n$ – Filippov conditions

$$\dot{x}(t) = \xi_k \in F(x(t_k)) \text{ (~ sampling)}$$

$$t \in [t_k, t_{k+1}],\ 0 < t_{k+1} - t_k \leq \tau$$

Over any time segment τ-solutions converge to Filippov solutions with $\tau \to 0$.

Proposition. With any $\{t_k\}$, $t_k \to \infty$, τ-solutions of *r*-sliding homogeneous controllers, linear asymptotically stable controllers, etc. converge into small vicinity of 0.

$$\| x \| \geq R \Rightarrow \| \dot{x} \| \tau / \| x \| = O(\tau)$$

Feasibility of fixed-time stable control-2

$\dot{x} \in F(x)$ is fixed-time or practically fixed-time stable.

Let $T_{enter}(R)$ = sup of the times needed for the solutions to enter $\|x\| \leq R$. $\Rightarrow \exists \lim_{R \to \infty} T_{enter}(R) \geq 0$

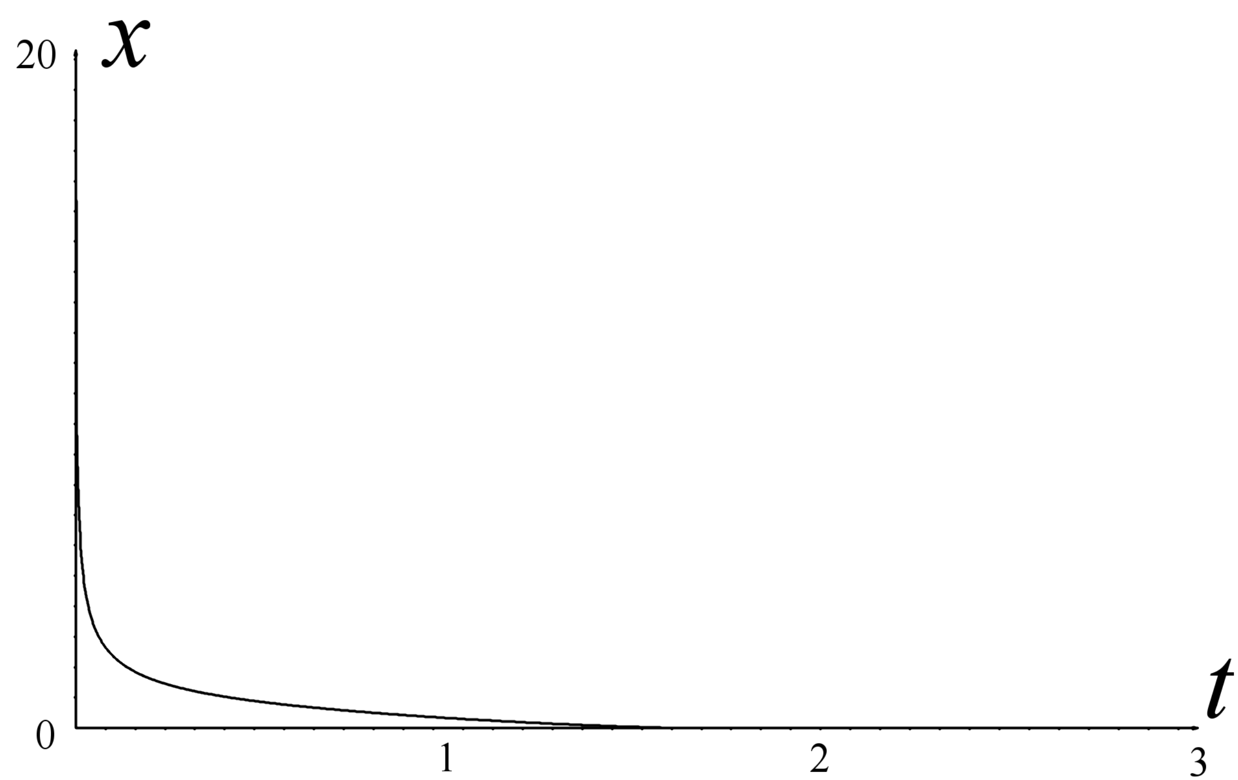
Usually $\lim_{R \to \infty} T_{enter}(R) = 0$ (Lyap. FxTS functions, uniform differentiators, homogeneity at infinity, etc.)

Theorem. $\lim_{R \to \infty} T_{enter}(R) = 0$. For any $\tau > 0$ for any large initial conditions there are τ-solutions which escape to infinity faster than any exponent.

Simulation example

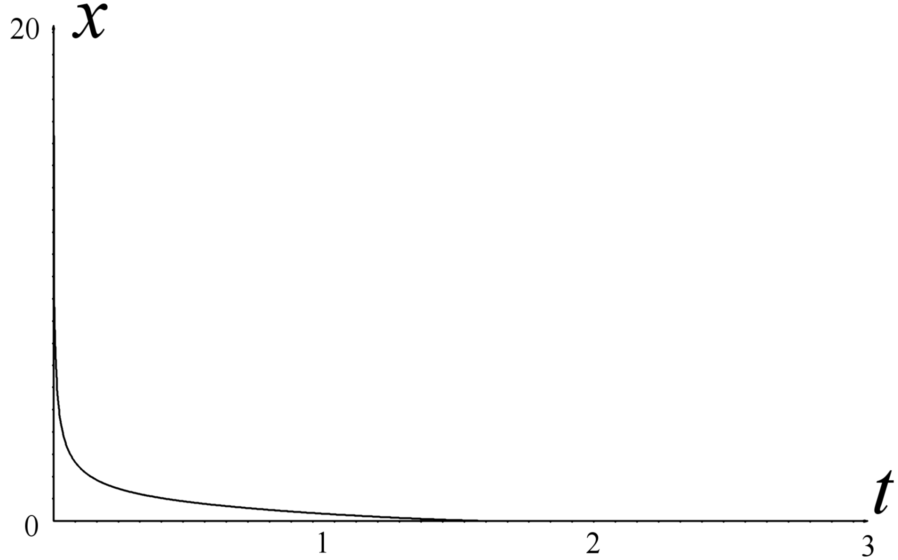
Classical scalar fixed-time stable system

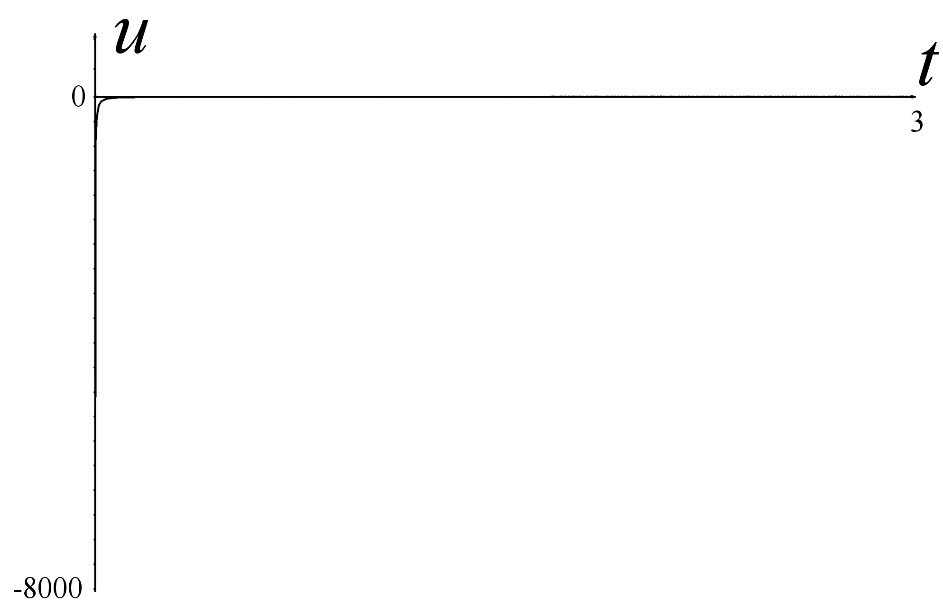
$$\dot{x} = u = -x^{1/3} - x^3$$



Fixed-time stabilization

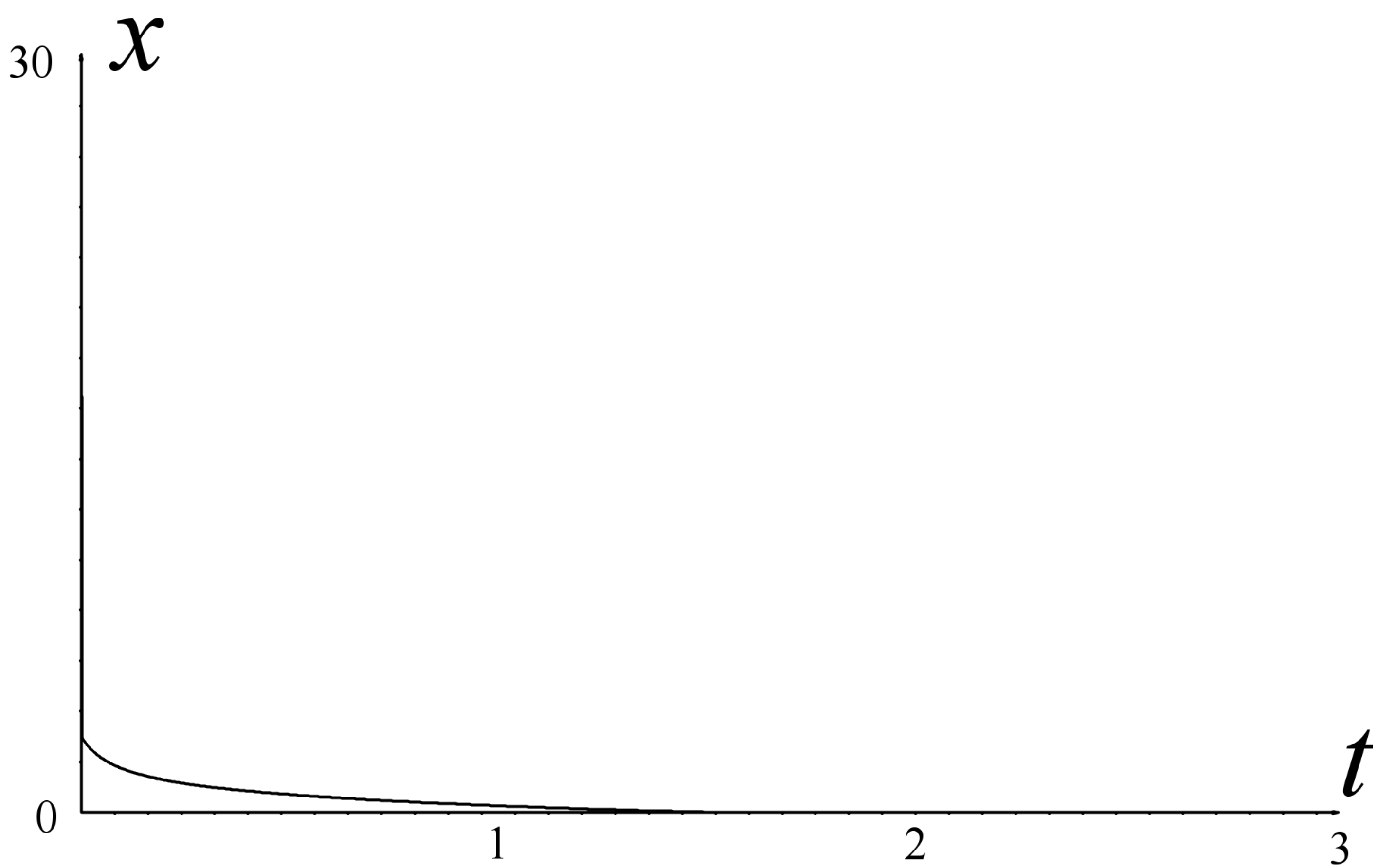
Initial values $x = 20,\ \tau = 10^{-3}$





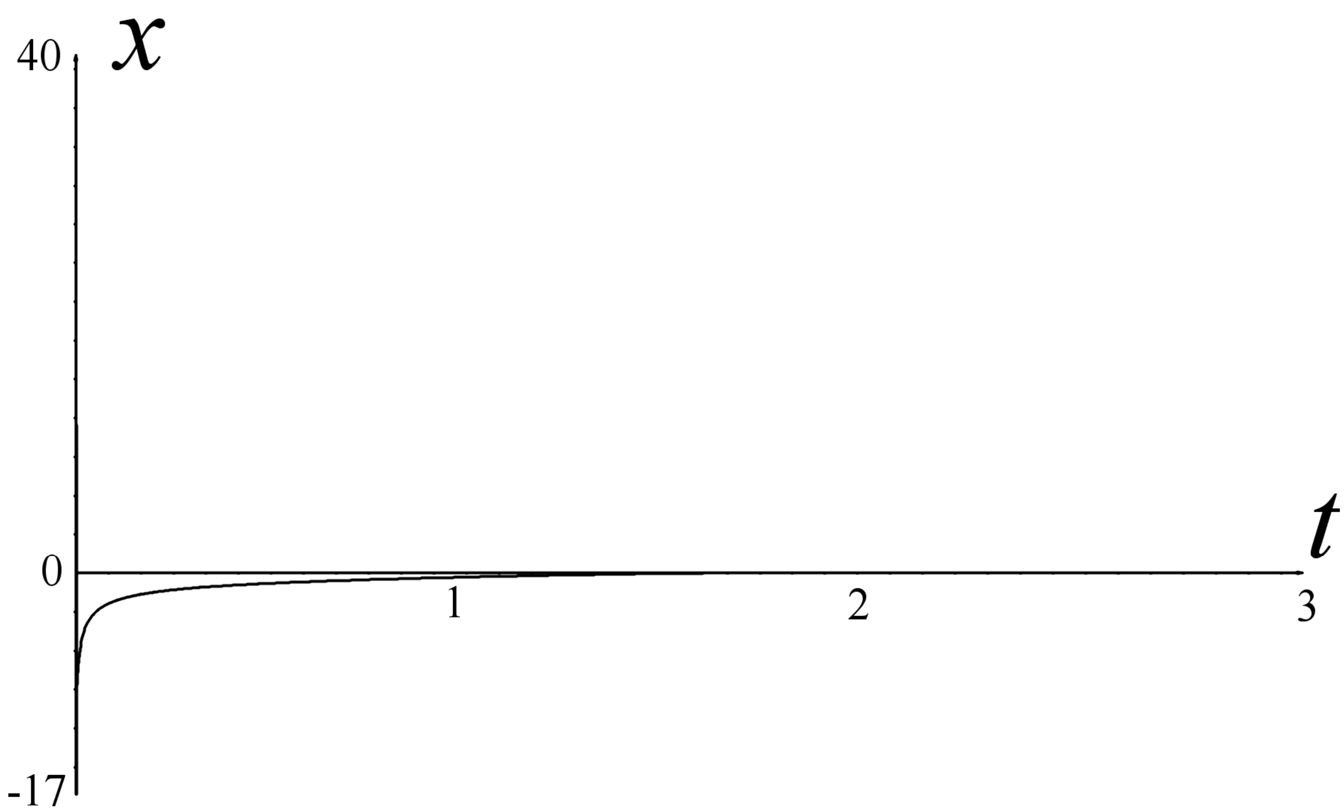
Accuracy: $|x| \leq 1.12 \cdot 10^{-5}$,

Initial value $x(0) = 30,\ \tau = 10^{-3}$



Accuracy: $|x| \le 1.12 \cdot 10^{-5}$

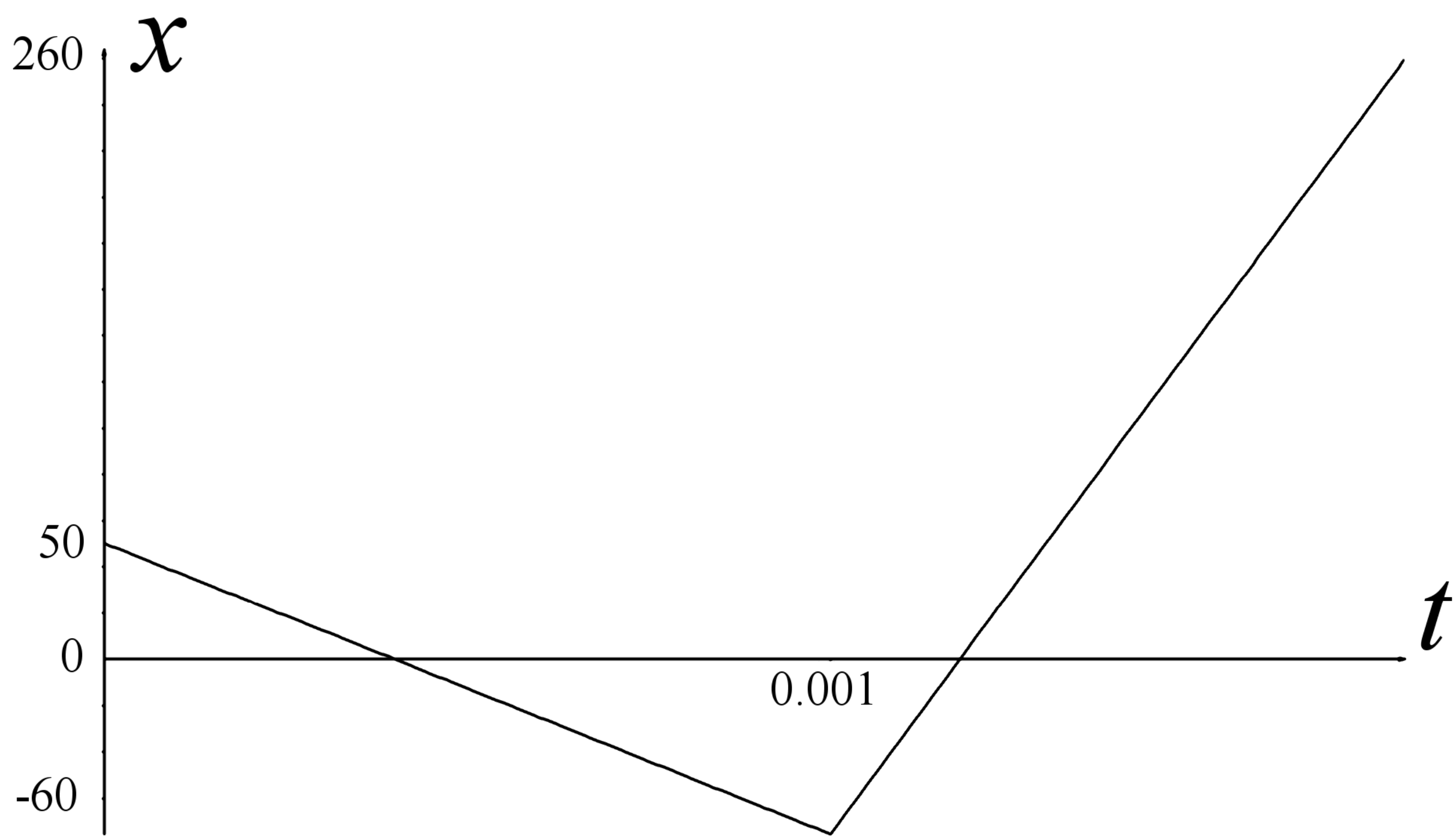
Initial value $x(0) = 40,\ \tau = 10^{-3}$. Overshoot!



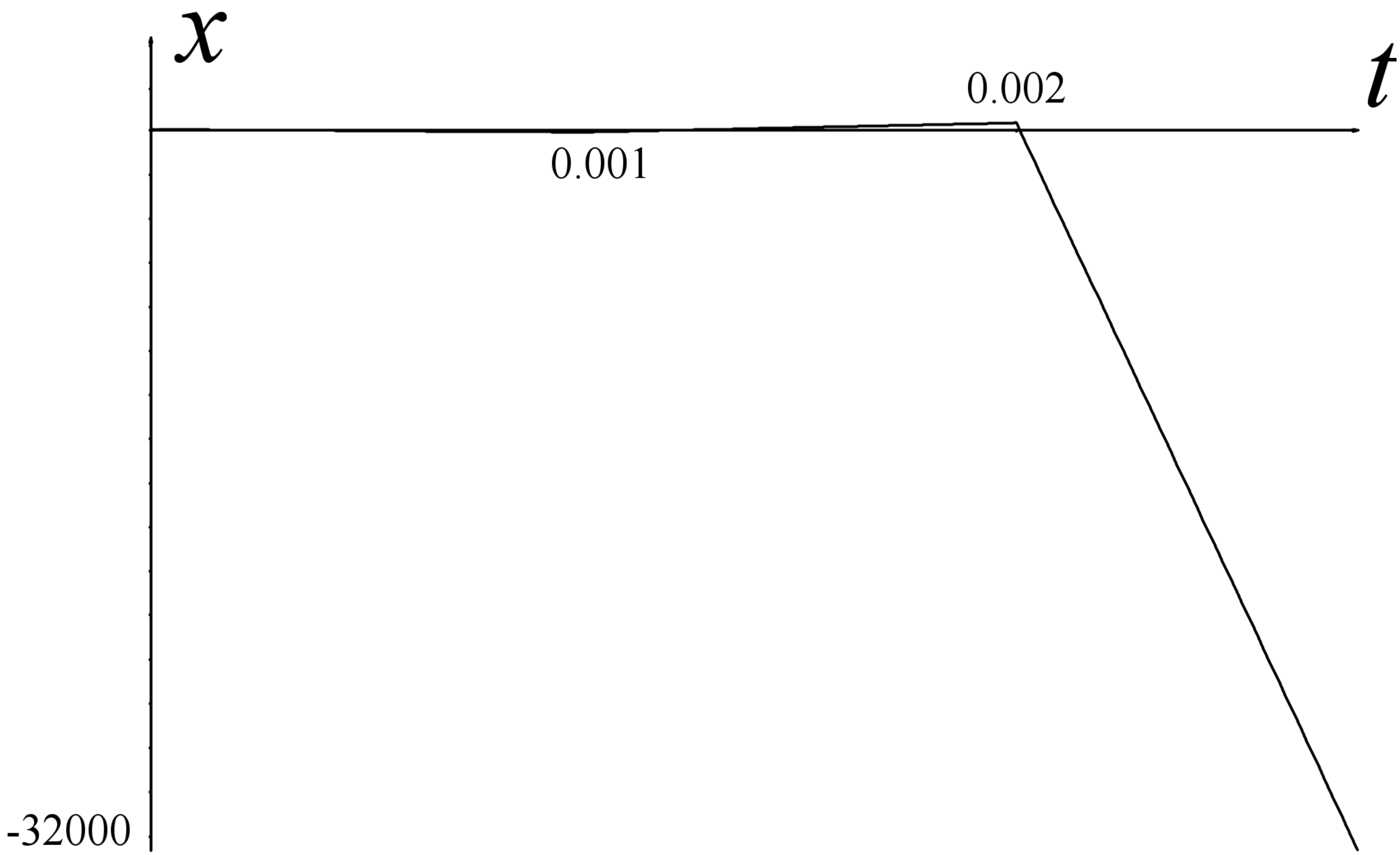
Accuracy: $|x| \leq 1.12 \cdot 10^{-5}$

Explosion with $x(0) = 50$, $\tau = 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}$.

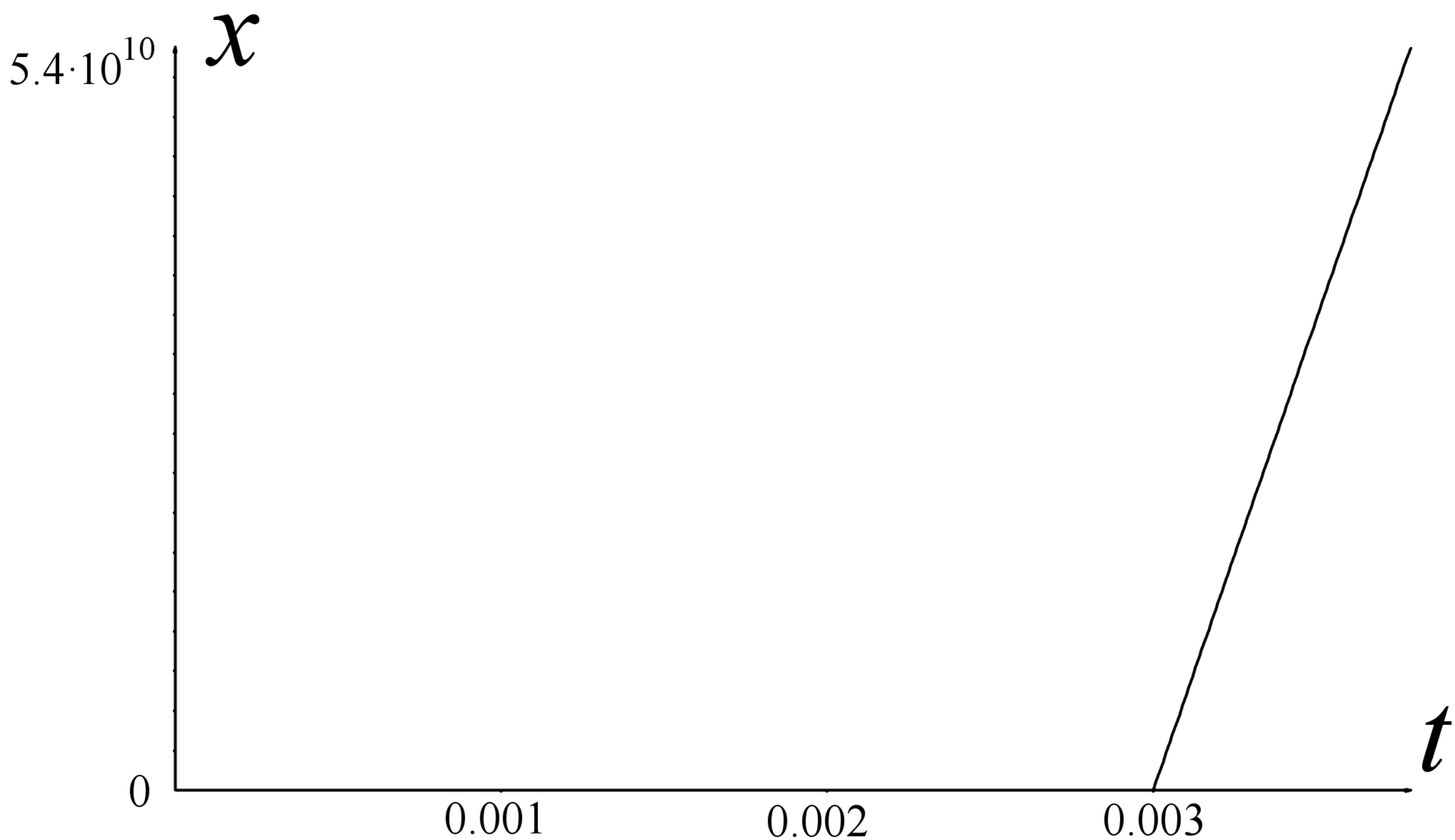
Results with $\tau = 10^{-3}$: 2 steps



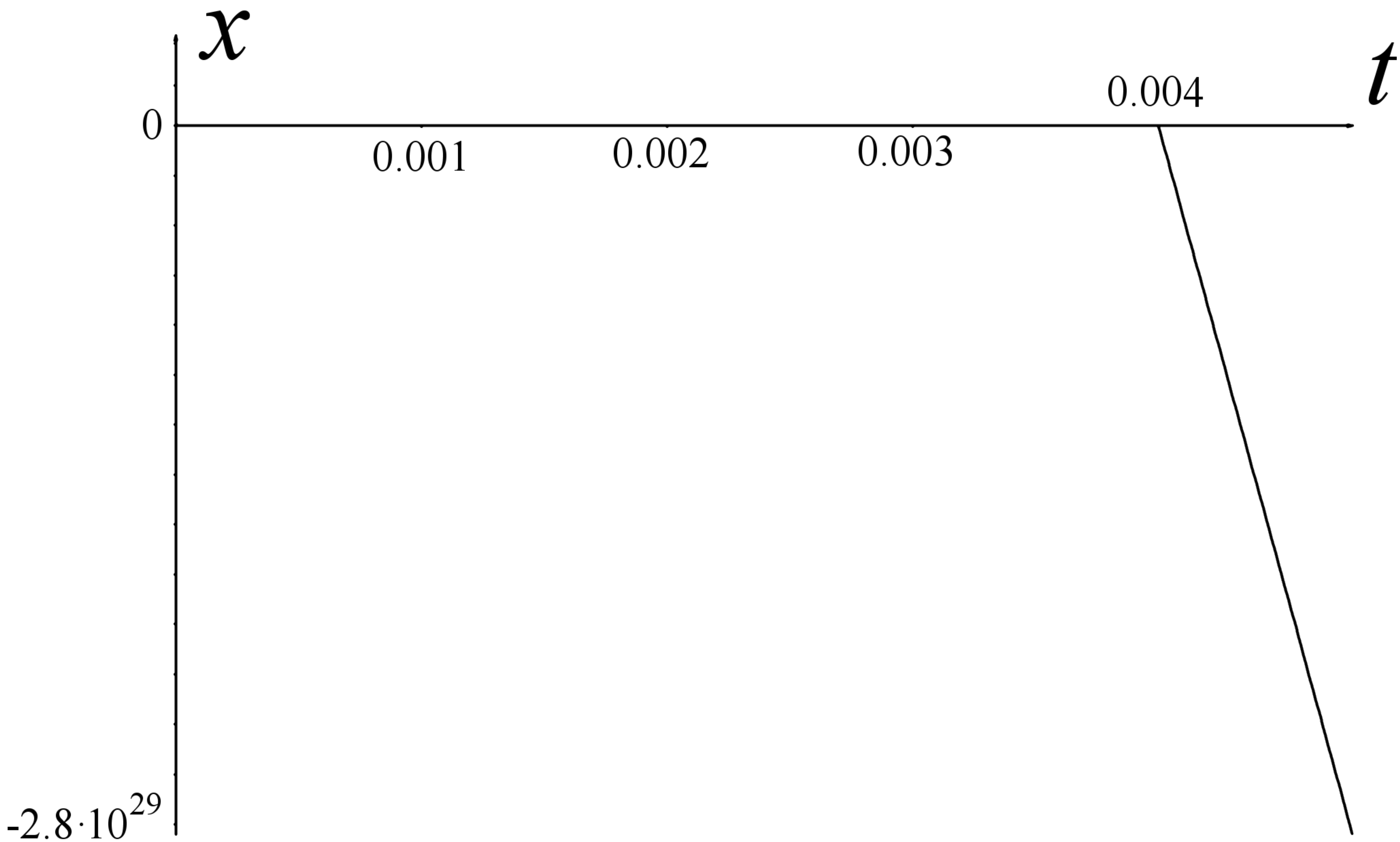
Results with $\tau = 10^{-3}$: 3 steps



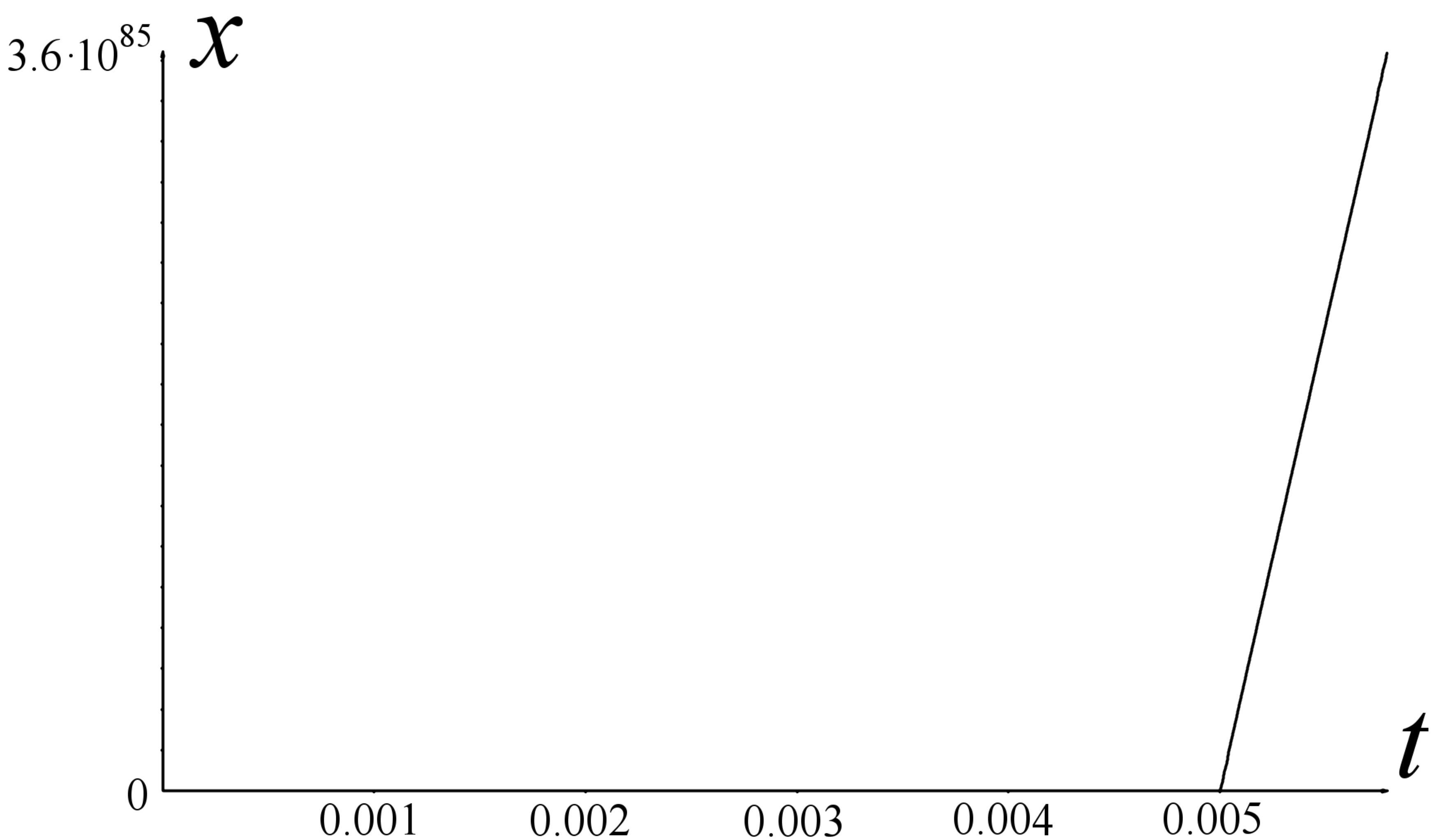
Results with $\tau = 10^{-3}$: 4 steps



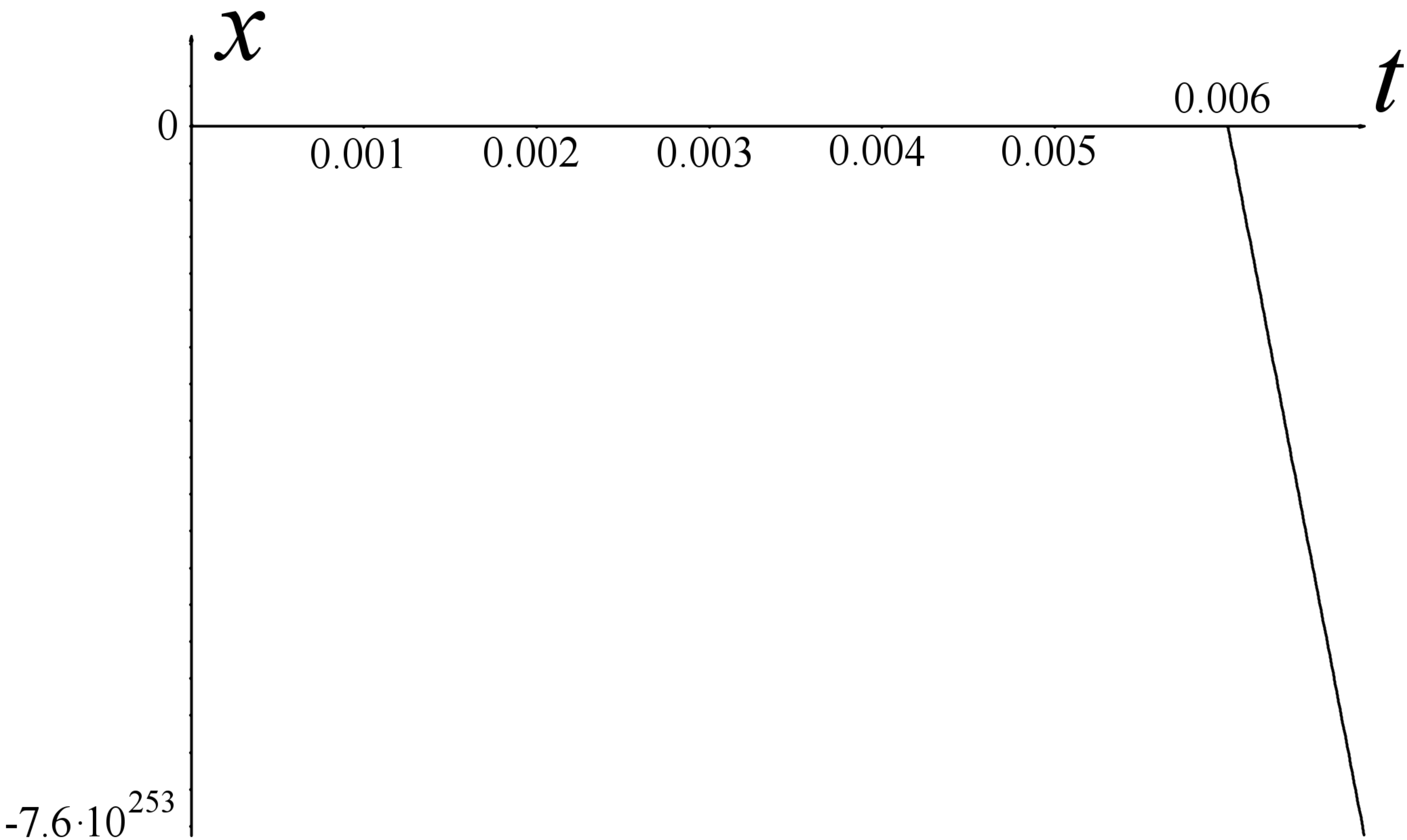
Results with $\tau = 10^{-3}$: 5 steps



Results with $\tau = 10^{-3}$: 6 steps



$\tau = 10^{-3}$: OVERFLOW at t = 0.007



Conclusions

- Computer realization of fixed-time stable systems over unbounded operational regions is not possible.
- Such systems are realizable for any bounded region of initial conditions, provided the sampling/integration step is taken small enough. Sampling step should be very carefully chosen.